

# Effect of different Dzyaloshinskii-Moriya interactions on entanglement in the Heisenberg XYZ chain

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In this paper, we study the thermal entanglement in a two-qubit Heisenberg XYZ system with different Dzyaloshinskii-Moriya (DM) couplings. We show that different DM coupling parameters have different influences on the entanglement and the critical temperature. In addition, we find that when  $J_i$  ( $i$ -component spin coupling interaction) is the largest spin coupling coefficient,  $D_i$  ( $i$ -component DM interaction) is the most efficient DM control parameter, which can be obtained by adjusting the direction of DM interaction.

PACS numbers: 03.67.Mn, 75.10.Jm, 03.67.Lx

## I. INTRODUCTION

Entanglement has been studied intensely in recent years due to its fascinating nonclassical feature and potential applications in quantum information processing [1]. As a simple system, Heisenberg model is an ideal candidate for the generation and the manipulation of entangled states. Many physical systems, such as nuclear spins [2], quantum dots [3], superconductor [4] and optical lattices [5], have been simulated by this model, and the Heisenberg interaction alone can be used for quantum computation by suitable coding [6]. Recently, the Heisenberg models, including Ising model [7], XY model [8], XXX model [9], XXZ model [10] and XYZ model [11, 12], have been intensively studied. Shan *et al.* investigated the effects of DM interaction, impurity and exchange couplings on entanglement in XY spin chain [13]. Aydinler *et al.* studied the thermal entanglement of a two-qutrit Ising system with DM interaction [14], they found that the control of entanglement can be optimized by utilizing competing effects of the magnetic field and the DM interaction. Wang *et al.* investigated the effects of the DM interaction and intrinsic decoherence on entanglement teleportation in the two-qubit XXX Heisenberg model [15].

In the above models, the influences of the z-component DM interaction (arising from spin-orbit coupling) and the external magnetic field on the entanglement have been discussed, but the DM coupling interactions along other directions have never been taken into account. Quite recently, we discussed the influences of x-component DM interaction on entanglement in Heisenberg XXZ model [16] and XYZ model [17]. To research further the differences between DM coupling interactions along different directions, in this paper, we generalize the special Heisenberg models to the generalized Heisenberg XYZ models with different DM interactions, and then analyze the different influences of  $D_x$  (x-component DM control parameter),  $D_y$  (y-component DM control parameter) and  $D_z$  (z-component DM control parameter) on the entanglement and the critical temperature. We

find that  $D_i$  is the most efficient DM control parameter when  $J_i$  is the largest spin coupling coefficient. Thus, according to the relation among  $J_i$  ( $i = x, y, z$ ), we can know which is the most efficient DM control parameter. In order to provide a detailed analytical and numerical analysis, here we take concurrence as a measure of entanglement [18]. The concurrence  $C$  ranges from 0 to 1,  $C = 0$  and  $C = 1$  indicate the vanishing entanglement and the maximal entanglement respectively. For a mixed state  $\rho$ , the concurrence of the state is  $C(\rho) = \max\{2\lambda_{\max} - \sum_{i=1}^4 \lambda_i, 0\}$ , where  $\lambda_i$ s are the positive square roots of the eigenvalues of the matrix  $R = \rho(\sigma^y \otimes \sigma^y) \rho^*(\sigma^y \otimes \sigma^y)$ , and the asterisk denotes the complex conjugate.

This paper is organized as follows. In Sec. II, we introduce the Heisenberg XYZ models with different DM interaction parameters, and give the analytical expressions of the concurrences. In Sec. III, we analyze the different influences of different DM control parameters ( $D_x$ ,  $D_y$  and  $D_z$ ) on the entanglement and the critical temperature. Finally, in Sec. IV a discussion concludes the paper.

## II. THE HEISENBERG XYZ MODELS WITH DIFFERENT DM INTERACTION PARAMETERS

### A. Heisenberg XYZ model with $D_x$

The Hamiltonian  $H$  for a two-qubit anisotropic Heisenberg XYZ chain with DM interaction parameter  $D_x$  is

$$H = J_x \sigma_1^x \sigma_2^x + J_y \sigma_1^y \sigma_2^y + J_z \sigma_1^z \sigma_2^z + D_x (\sigma_1^y \sigma_2^z - \sigma_1^z \sigma_2^y), \quad (1)$$

where  $J_i$  ( $i = x, y, z$ ) are the real coupling coefficients,  $D_x$  is the x-component DM coupling parameter, and  $\sigma^i$  ( $i = x, y, z$ ) are the Pauli matrices. The coupling constants  $J_i > 0$  corresponds to the antiferromagnetic case, and  $J_i < 0$  corresponds to the ferromagnetic case. This model can be reduced to some special Heisenberg models by changing  $J_i$ . Parameters  $J_i$  and  $D_x$  are dimensionless.

In the standard basis  $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$ , the Hamilto-

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nian (1) can be rewritten as

$$H = \begin{pmatrix} J_z & iD_x & -iD_x & J_x - J_y \\ -iD_x & -J_z & J_x + J_y & iD_x \\ iD_x & J_x + J_y & -J_z & -iD_x \\ J_x - J_y & -iD_x & iD_x & J_z \end{pmatrix}. \quad (2)$$

By calculating, we can obtain the eigenstates of  $H$ :

$$|\Psi_1\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle), \quad (3a)$$

$$|\Psi_2\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle), \quad (3b)$$

$$|\Psi_3\rangle = \frac{1}{\sqrt{2}}(\sin\theta_1|00\rangle - i\cos\theta_1|01\rangle + i\cos\theta_1|10\rangle - \sin\theta_1|11\rangle), \quad (3c)$$

$$|\Psi_4\rangle = \frac{1}{\sqrt{2}}(\sin\theta_2|00\rangle + i\cos\theta_2|01\rangle - i\cos\theta_2|10\rangle - \sin\theta_2|11\rangle), \quad (3d)$$

with corresponding eigenvalues:

$$E_{1,2} = J_x \pm J_y \mp J_z, \quad (4a)$$

$$E_{3,4} = -J_x \pm w, \quad (4b)$$

where  $\theta_{1,2} = \arctan(\frac{2D_x}{w \mp J_y \mp J_z})$ , and  $w = \sqrt{4D_x^2 + (J_y + J_z)^2}$ . The system state at thermal equilibrium (thermal state) is  $\rho(T) = \frac{\exp(\frac{-H}{K_B T})}{Z}$ , where  $Z = \text{Tr}[\exp(\frac{-H}{K_B T})]$  is the partition function of the system,  $H$  is the system Hamiltonian,  $T$  is the temperature and  $K_B$  is the Boltzmann constant which we take equal to 1 for simplicity. Thus, in the above standard basis, we can get the following analytical expression of the density matrix  $\rho(T)$ :

$$\rho(T) = \begin{pmatrix} m_1 & q & q^* & m_2 \\ q^* & n_1 & n_2 & q \\ q & n_2 & n_1 & q^* \\ m_2 & q^* & q & m_1 \end{pmatrix}, \quad (5)$$

where

$$m_{1,2} = \frac{1}{2Z}(e^{-\frac{E_2}{T}} \pm e^{-\frac{E_3}{T}} \sin^2\theta_1 \pm e^{-\frac{E_4}{T}} \sin^2\theta_2),$$

$$n_{1,2} = \frac{1}{2Z}(e^{-\frac{E_1}{T}} \pm e^{-\frac{E_3}{T}} \cos^2\theta_1 \pm e^{-\frac{E_4}{T}} \cos^2\theta_2),$$

$$q = \frac{i}{2Z}(e^{-\frac{E_3}{T}} \sin\theta_1 \cos\theta_1 - e^{-\frac{E_4}{T}} \sin\theta_2 \cos\theta_2).$$

After straightforward calculations, the positive square roots of the eigenvalues of the matrix  $R = \rho(\sigma^y \otimes \sigma^y)\rho^*(\sigma^y \otimes \sigma^y)$  can be expressed as:

$$\lambda_{1,2} = \frac{1}{Z}e^{\frac{J_x \pm w}{T}}, \quad (6a)$$

$$\lambda_{3,4} = \frac{1}{Z}e^{\frac{-J_x \pm J_y \mp J_z}{T}}, \quad (6b)$$

where  $Z = 2e^{\frac{-J_x}{T}} \cosh(\frac{J_y - J_z}{T}) + 2e^{\frac{J_x}{T}} \cosh(\frac{w}{T})$ . Thus, the concurrence of this system can be expressed as [18]:

$$C = \begin{cases} \max\{|\lambda_1 - \lambda_3| - \lambda_2 - \lambda_4, 0\}, & \text{if } J_y > J_z, \\ \max\{|\lambda_1 - \lambda_4| - \lambda_2 - \lambda_3, 0\}, & \text{if } J_y \leq J_z. \end{cases} \quad (7)$$

which is consistent with the results in Ref. [16] for  $J_x = J_y$ .

## B. Heisenberg XYZ model with $D_y$

Here we consider the case of the two-qubit anisotropic Heisenberg XYZ chain with y-component DM parameter  $D_y$ . The Hamiltonian is

$$H' = J_x \sigma_1^x \sigma_2^x + J_y \sigma_1^y \sigma_2^y + J_z \sigma_1^z \sigma_2^z + D_y (\sigma_1^z \sigma_2^x - \sigma_1^x \sigma_2^z), \quad (8)$$

where  $D_y$  is the y-component DM coupling parameter, which is also dimensionless.

In the standard basis  $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$ , the Hamiltonian (8) can be rewritten as

$$H' = \begin{pmatrix} J_z & D_y & -D_y & J_x - J_y \\ D_y & -J_z & J_x + J_y & D_y \\ -D_y & J_x + J_y & -J_z & -D_y \\ J_x - J_y & D_y & -D_y & J_z \end{pmatrix}. \quad (9)$$

Similarly, by calculating, we can obtain the eigenstates of  $H'$ :

$$|\Psi'_1\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle), \quad (10a)$$

$$|\Psi'_2\rangle = \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle), \quad (10b)$$

$$|\Psi'_3\rangle = \frac{1}{\sqrt{2}}(\sin\phi_1|00\rangle - \cos\phi_1|01\rangle + \cos\phi_1|10\rangle + \sin\phi_1|11\rangle), \quad (10c)$$

$$|\Psi'_4\rangle = \frac{1}{\sqrt{2}}(\sin\phi_2|00\rangle - \cos\phi_2|01\rangle + \cos\phi_2|10\rangle + \sin\phi_2|11\rangle), \quad (10d)$$

with corresponding eigenvalues:

$$E'_{1,2} = J_y \pm J_x \mp J_z, \quad (11a)$$

$$E'_{3,4} = -J_y \pm w', \quad (11b)$$

where  $\phi_{1,2} = \arctan(\frac{2D_y}{J_x + J_z \mp w'})$ , and  $w' = \sqrt{4D_y^2 + (J_x + J_z)^2}$ . In the above standard basis, the density matrix  $\rho'(T)$  has the following form:

$$\rho'(T) = \begin{pmatrix} m'_1 & -q' & q' & m'_2 \\ -q' & n'_1 & n'_2 & -q' \\ q' & n'_2 & n'_1 & q' \\ m'_2 & -q' & q' & m'_1 \end{pmatrix}, \quad (12)$$

where

$$m'_{1,2} = \frac{1}{2Z'} (\pm e^{-\frac{E'_2}{T}} + e^{-\frac{E'_3}{T}} \sin^2 \phi_1 + e^{-\frac{E'_4}{T}} \sin^2 \phi_2),$$

$$n'_{1,2} = \frac{1}{2Z'} (e^{-\frac{E'_1}{T}} \pm e^{-\frac{E'_2}{T}} \cos^2 \phi_1 \pm e^{-\frac{E'_3}{T}} \cos^2 \phi_2),$$

$$q' = \frac{1}{2Z'} (e^{-\frac{E'_3}{T}} \sin \phi_1 \cos \phi_1 + e^{-\frac{E'_4}{T}} \sin \phi_2 \cos \phi_2).$$

Then the positive square roots of the eigenvalues of the matrix  $R' = \rho'(\sigma^y \otimes \sigma^y) \rho'^*(\sigma^y \otimes \sigma^y)$  can be obtained

$$\lambda'_{1,2} = \frac{1}{Z'} e^{-\frac{J_y \pm J_x \mp J_z}{T}}, \quad (13a)$$

$$\lambda'_{3,4} = \frac{1}{Z'} e^{\frac{J_y \pm w'}{T}}, \quad (13b)$$

where  $Z' = 2e^{-\frac{J_y}{T}} \cosh(\frac{J_x - J_z}{T}) + 2e^{\frac{J_y}{T}} \cosh(\frac{w'}{T})$ . Thus, the concurrence of this system can be expressed as:

$$C = \begin{cases} \max\{|\lambda'_1 - \lambda'_3| - \lambda'_2 - \lambda'_4, 0\}, & \text{if } J_x > J_z, \\ \max\{|\lambda'_2 - \lambda'_3| - \lambda'_1 - \lambda'_4, 0\}, & \text{if } J_x \leq J_z. \end{cases} \quad (14)$$

### C. Heisenberg XYZ model with $D_z$

The Hamiltonian  $H''$  of a two-qubit anisotropic Heisenberg XYZ chain with z-component DM parameter  $D_z$  is

$$H'' = J_x \sigma_1^x \sigma_2^x + J_y \sigma_1^y \sigma_2^y + J_z \sigma_1^z \sigma_2^z + D_z (\sigma_1^x \sigma_2^y - \sigma_1^y \sigma_2^x), \quad (15)$$

where  $D_z$  is the z-component DM coupling parameter, which is also dimensionless.

Using the similar process, we can get the eigenstates of  $H''$ :

$$|\Psi''_{1,2}\rangle = \frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle), \quad (16a)$$

$$|\Psi''_{3,4}\rangle = \frac{1}{\sqrt{2}}(|01\rangle \pm \chi|10\rangle), \quad (16b)$$

with corresponding eigenvalues:

$$E''_{1,2} = J_z \pm J_x \mp J_y, \quad (17a)$$

$$E''_{3,4} = -J_z \pm w'', \quad (17b)$$

where  $\chi = \frac{J_x + J_y - 2iD_z}{\sqrt{4D_z^2 + (J_x + J_y)^2}}$ , and  $w'' = \sqrt{4D_z^2 + (J_x + J_y)^2}$ . Similarly, we can get the analytical expressions of  $\rho(T)''$  and  $R''$ , but we do not list them because of the complexity. After straightforward calculations, the positive square roots of the eigenvalues of  $R'' = \rho''(\sigma^y \otimes \sigma^y) \rho''^*(\sigma^y \otimes \sigma^y)$  can be expressed as:

$$\lambda''_{1,2} = \frac{1}{Z''} e^{\frac{J_z \pm w''}{T}}, \quad (18a)$$

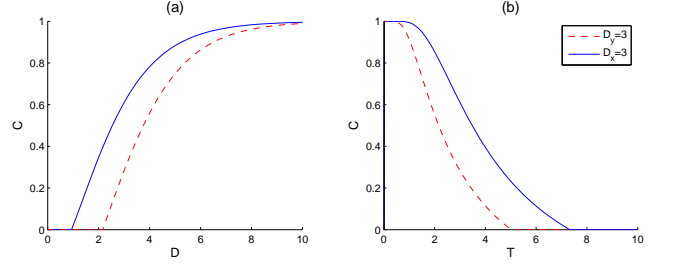


FIG. 1: (Color online) (a) The concurrence is plotted versus  $D_x$  (blue solid line) and  $D_y$  (red dashed line) for  $T = 3$ . (b) The concurrence is plotted as a function of the temperature  $T$  for  $D_x = 3$  (blue solid line) and  $D_y = 3$  (red dashed line). Here the coupling constants  $J_x = 0.2$ ,  $J_y = -1$  and  $J_z = -0.5$ .

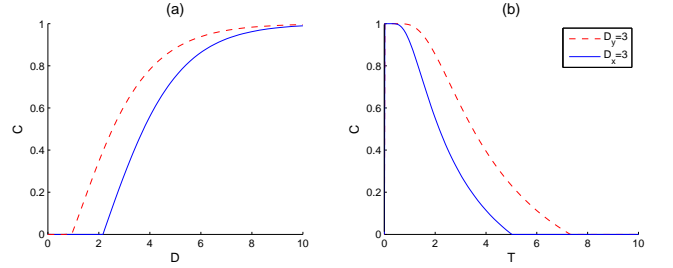


FIG. 2: (Color online) (a) The concurrence is plotted versus  $D_x$  (blue solid line) and  $D_y$  (red dashed line) for  $T = 3$ . (b) The concurrence is plotted as a function of the temperature  $T$  for  $D_x = 3$  (blue solid line) and  $D_y = 3$  (red dashed line). Here the coupling constants  $J_x = -1$ ,  $J_y = 0.2$  and  $J_z = -0.5$ .

$$\lambda''_{3,4} = \frac{1}{Z''} e^{-\frac{J_z \pm J_x \mp J_y}{T}}, \quad (18b)$$

with  $Z'' = 2e^{-\frac{J_z}{T}} \cosh(\frac{J_x - J_y}{T}) + 2e^{\frac{J_z}{T}} \cosh(\frac{w''}{T})$ . Thus, the concurrence of this system can be written as:

$$C = \begin{cases} \max\{|\lambda''_1 - \lambda''_3| - \lambda''_2 - \lambda''_4, 0\}, & \text{if } J_x > J_y, \\ \max\{|\lambda''_1 - \lambda''_4| - \lambda''_2 - \lambda''_3, 0\}, & \text{if } J_x \leq J_y. \end{cases} \quad (19)$$

which is also consistent with the results in Ref. [16] when  $J_x = J_y$ .

From Eqs. (7), (14) and (19), one can see that when  $J_x = J_y$ , there is the same entanglement for  $D_x = D_y$ ; when  $J_y = J_z$ , there is the same entanglement for  $D_y = D_z$ ; and when  $J_x = J_z$ , there is the same entanglement for  $D_x = D_z$ . So when  $J_x = J_y = J_z$ , there is also the same entanglement for the same values of DM interaction parameters ( $D_x$ ,  $D_y$  and  $D_z$ ).

## III. THE COMPARISON BETWEEN THE DIFFERENT DM CONTROL PARAMETERS IN HEISENBERG XYZ MODEL

### A. The comparison between $D_x$ and $D_y$

In Heisenberg XYZ model, we have analyzed all kinds of spin coupling coefficients satisfying  $J_x > J_y$ . For simplicity,

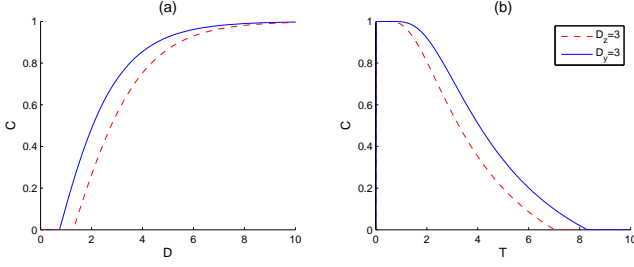


FIG. 3: (Color online) (a) The concurrence is plotted versus  $D_y$  (blue solid line) and  $D_z$  (red dashed line) for  $T = 3$ . (b) The concurrence is plotted as a function of the temperature  $T$  for  $D_y = 3$  (blue solid line) and  $D_z = 3$  (red dashed line). Here the coupling constants  $J_x = -0.5$ ,  $J_y = 1$  and  $J_z = 0.2$ .

here we choose one set of spin coupling coefficients satisfying  $J_x > J_y$ , and plot Fig. 1 to demonstrate the properties of different DM parameters. In Fig. 1(a), we find the entanglement increases with the increase of DM coupling parameter. Furthermore, the critical value of  $D_x$  is smaller than  $D_y$ , and  $D_x$  has more entanglement for  $D_x = D_y$ . In Fig. 1(b), we find that increasing temperature will decrease the entanglement, and  $D_x$  has a higher critical temperature than the same  $D_y$ , so that the entanglement can exist at higher temperatures for  $D_x$ .

Similarly, we have analyzed various spin coupling coefficients satisfying  $J_x < J_y$ . For simplicity, we choose one set of coupling coefficients satisfying  $J_x < J_y$ , and plot Fig. 2 to demonstrate the properties of different DM parameters. In Fig. 2(a), it is shown that increasing the DM coupling parameter can enhance the entanglement. Besides, for a certain temperature, the critical value of  $D_y$  is smaller than  $D_x$ , and  $D_y$  has more entanglement for  $D_x = D_y$ . In Fig. 2(b), it is easy to see that  $D_y$  has a higher critical temperature than the same  $D_x$ , so that the entanglement can exist at higher temperatures for  $D_y$ .

Thus, the x-component parameter  $D_x$  has a smaller critical value, higher critical temperature and more entanglement than the same y-component parameter  $D_y$  for  $J_x > J_y$ , and  $D_y$  has a smaller critical value, higher critical temperature and more entanglement than the same  $D_x$  for  $J_x < J_y$ .

#### B. The comparison between $D_y$ and $D_z$

Similarly, for  $J_y > J_z$  case, Fig. 3 is plotted to show the properties of different DM parameters in Heisenberg XYZ model. In Fig. 3(a), we can see that the y-component parameter  $D_y$  has a smaller critical value, and more entanglement for  $D_y = D_z$ . In Fig. 3(b), we can see that  $D_y$  has a higher critical temperature than the same  $D_z$ , so that the entanglement can exist at higher temperatures for  $D_y$ .

Contrarily, for  $J_y < J_z$  case, the concurrence versus different parameters is shown in Fig. 4. In Fig. 4(a), it is easy to see that the z-component parameter  $D_z$  has a smaller critical value, and more entanglement for  $D_y = D_z$ . In Fig. 4(b), it is easy to see that  $D_z$  has a higher critical temperature than the same  $D_y$ , so that the entanglement can also exist at higher

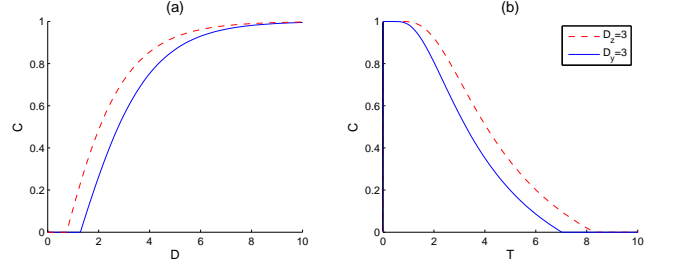


FIG. 4: (Color online) (a) The concurrence is plotted versus  $D_y$  (blue solid line) and  $D_z$  (red dashed line) for  $T = 3$ . (b) The concurrence is plotted as a function of the temperature  $T$  for  $D_y = 3$  (blue solid line) and  $D_z = 3$  (red dashed line). Here the coupling constants  $J_x = -0.5$ ,  $J_y = 0.2$  and  $J_z = 1$ .

temperatures for  $D_z$ .

Thus, the y-component parameter  $D_y$  has a smaller critical value, higher critical temperature and more entanglement than the same z-component parameter  $D_z$  for  $J_y > J_z$ , and  $D_z$  has a smaller critical value, higher critical temperature and more entanglement than the same  $D_y$  for  $J_y < J_z$ .

#### C. The comparison between $D_x$ and $D_z$

Here, for  $J_x > J_z$  case, we plot Fig. 5 to illustrate the properties of different DM parameters in Heisenberg XYZ model. In Fig. 5(a),  $D_x$  has a smaller critical value and more entanglement for  $D_x = D_z$ . In Fig. 5(b),  $D_x$  has a higher critical temperature than the same  $D_z$ , so that the entanglement can exist at higher temperatures for  $D_x$ .

For  $J_x < J_z$  case, we plot Fig. 6 to illustrate the properties of different DM parameters. In Fig. 6(a), for a certain temperature,  $D_z$  has a smaller critical value and more entanglement for  $D_x = D_z$ . In Fig. 6(b),  $D_z$  has a higher critical temperature than the same  $D_x$ , so that the entanglement can also exist at higher temperatures for  $D_z$ .

Thus, the x-component parameter  $D_x$  has a smaller critical value, higher critical temperature and more entanglement than the same z-component parameter  $D_z$  for  $J_x > J_z$ , and  $D_z$  has a smaller critical value, higher critical temperature and more entanglement than the same  $D_x$  for  $J_x < J_z$ .

The above results indicate that when  $J_i$  is the largest spin coupling coefficient, the i-component DM control parameter  $D_i$  has the smallest critical value, the highest critical temperature and the most entanglement. So according to the relation among spin coupling coefficients, we can know which is the most efficient DM control parameter.

#### IV. DISCUSSION

We have investigated the thermal entanglement in a two-qubit Heisenberg XYZ system with different Dzyaloshinskii-Moriya (DM) couplings. We find that different DM interaction parameters ( $D_x$ ,  $D_y$  and  $D_z$ ) have different influences on the entanglement and the critical temperature. When

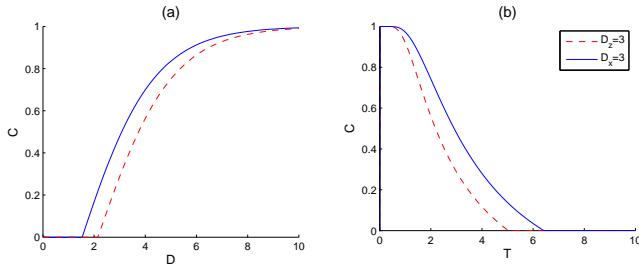


FIG. 5: (Color online) (a) The concurrence is plotted versus  $D_x$  (blue solid line) and  $D_z$  (red dashed line) for  $T = 3$ . (b) The concurrence is plotted as a function of the temperature  $T$  for  $D_x = 3$  (blue solid line) and  $D_z = 3$  (red dashed line). Here the coupling constants  $J_x = -0.2$ ,  $J_y = 0.3$  and  $J_z = -1$ .

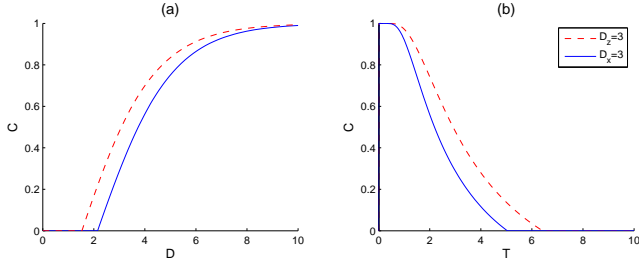


FIG. 6: (Color online) (a) The concurrence is plotted versus  $D_x$  (blue solid line) and  $D_z$  (red dashed line) for  $T = 3$ . (b) The concurrence is plotted as a function of the temperature  $T$  for  $D_x = 3$  (blue solid line) and  $D_z = 3$  (red dashed line). Here the coupling constants  $J_x = -1$ ,  $J_y = 0.3$  and  $J_z = -0.2$ .

$J_i$  ( $i = x, y, z$ ) is the largest spin coupling coefficient, the  $i$ -component DM interaction  $D_i$  ( $i = x, y, z$ ) has the smallest critical value, the highest critical temperature and the most entanglement. In addition, when  $J_x = J_y = J_z$ , there is the same entanglement for the same values of DM interaction parameters ( $D_x$ ,  $D_y$  and  $D_z$ ). Thus, according to the relation among spin coupling coefficients ( $J_x$ ,  $J_y$  and  $J_z$ ), the most efficient DM control parameter can be obtained by adjusting the direction of DM interaction.

### Acknowledgments

This work is supported by National Natural Science Foundation of China (NSFC) under Grant Nos: 60678022 and 10704001, the Specialized Research Fund for the Doctoral Program of Higher Education under Grant No. 20060357008, Anhui Provincial Natural Science Foundation under Grant No. 070412060, and the Key Program of the Education Department of Anhui Province under Grant No. KJ2008A28ZC.

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